Situation 01: Sine 32°

Prepared at Penn State

Mid-Atlantic Center for Mathematics Teaching and Learning

Pat Wilson, Heather Godine, and Jeanne Shimizu

June 29, 2005

Prompt

This vignette takes place in a high school mathematics classroom. The class is being taught by a student teacher.

After having completed a discussion on special right triangles (30°-60°-90° and 45°-45°-90°), the student teacher turned the class discussion to right triangle trigonometry. The student teacher showed students how to calculate the sine of various angles using the calculator.

A student then asked, “How could I calculate sin (32°) if I do not have a calculator?

Mathematical Foci

Mathematical Focus 1

The sine function is not a linear function but we can use a linear function to approximate the function over sufficiently small intervals. This approach makes use of the notion that a line can be used to approximate a differentiable function for points that are close together. That is, for $(x_1, f(x_1))$ and $(x_2, f(x_2))$ and $x_1 < x < x_2$, if $x_1 - x_2$ is close to 0, $f(x)$ can be approximated by a line. In essence, we are using a secant line to approximate the curve.

$\mathrm{Sin}(32°)$ can be approximated using linear interpolation with $\sin(30°)$ and $\sin(45°)$. Figure 1 shows that the sine function is approximately linear between points A and B, where the coordinates of A are $(30°, \sin 30°)$ and the coordinates of B are $(45°, \sin 45°)$

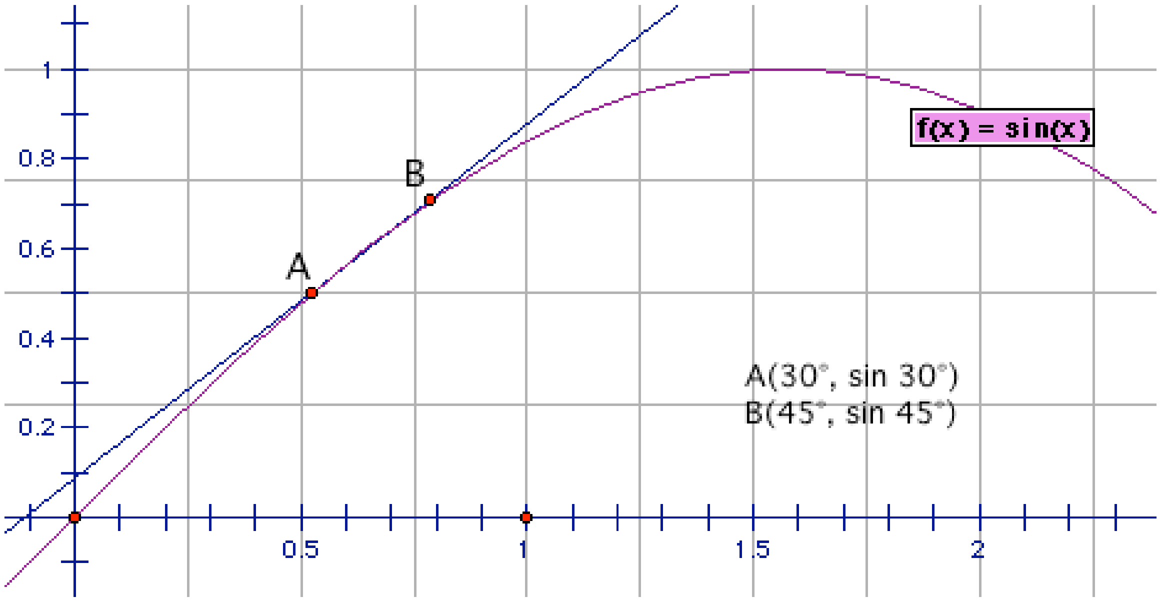


Figure 1.

Figure 2 shows that point C whose coordinates are $(32°, \sin 32°)$ can be approximated by point D, a point on secant line AB.

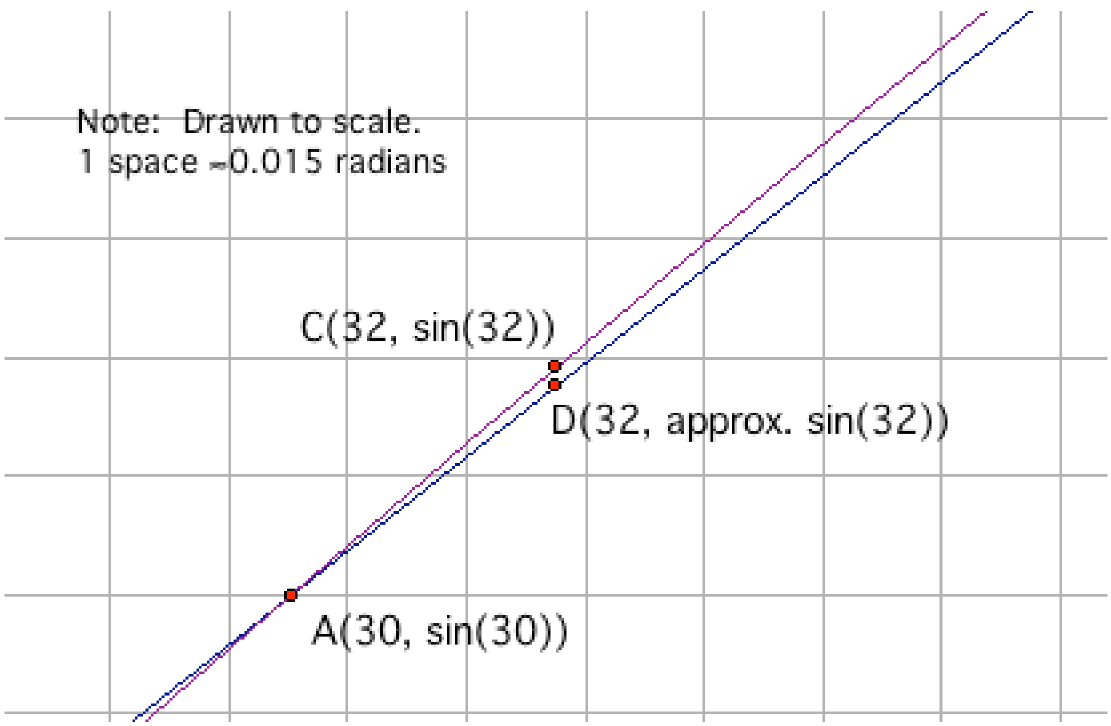


Figure 2.

To approximate sin(32°), we can use sin(45°) and sin(30°) as follows:

θ   <u>sin (θ)</u>

30	0.500
32	y
45	0.707

$$\frac{32-30}{45-30} \approx \frac{y-0.500}{0.707-0.500}$$

$$\frac{2}{15} \approx \frac{y-0.500}{0.207}$$

$$y \approx \frac{(2)(0.207)}{15} + 0.500 \approx 0.5276$$

So, $\sin(32°) \approx 0.5276$ (Note: This is close to the TI-92 approximation, $\sin(32°) \approx 0.5299$)

This method of approximation is connected to the concept of similar triangles. Figure 3 illustrates the triangles being used.

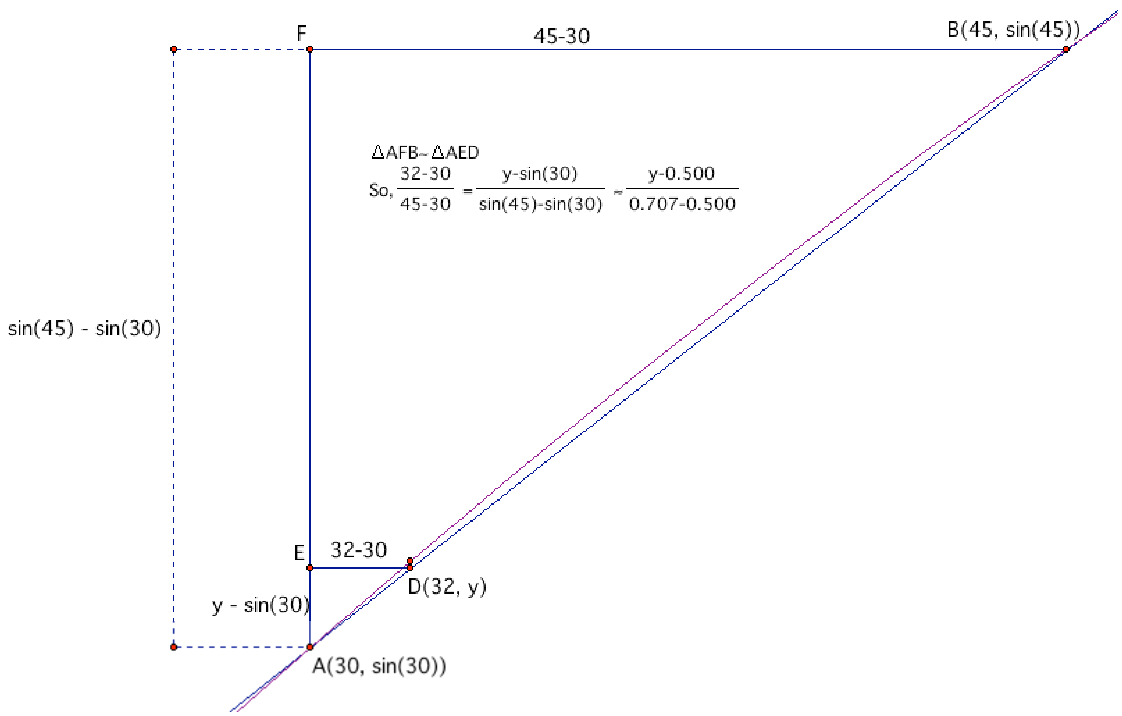


Figure 3

The value calculated relates to the equation of secant line through $(30°,\sin(30°))$ and $(45°,\sin(45°))$. We can approximate the coefficients for an equation of the secant line and then use the resulting equation to calculate the value of y for x=32:

$$y-0.500=\frac{0.707-0.500}{45-30}(x-30)$$

$$y=0.0138x+0.086$$

$$f(32)\approx 0.0138(32)+0.086=0.5276$$

Mathematical Focus 2

$Sin(32°)$ can be approximated using linear approximation and differentials.

This approach makes use of the notion that a tangent line can be used to approximate a differentiable function at a nearby point. That is, given $(a,f(a))$, *predict the value of* $f(x)$ at a nearby point, $a+dx$. When *dx* is small, the value of $f(a+dx)$ *and the value of the tangent line at* $a+dx$ *will be very close. To make use of differentials, we must use radian measure.*

$$32° \text{ is equivalent to } \frac{32\pi}{180} \text{ radians.}$$

$$\frac{32\pi}{180} \approx 0.5585$$

This focus is based on a geometric interpretation of differentials dx and dy and their relation to Δx and Δy where a tangent line can be used to approximate $f(x)$ near a given value.

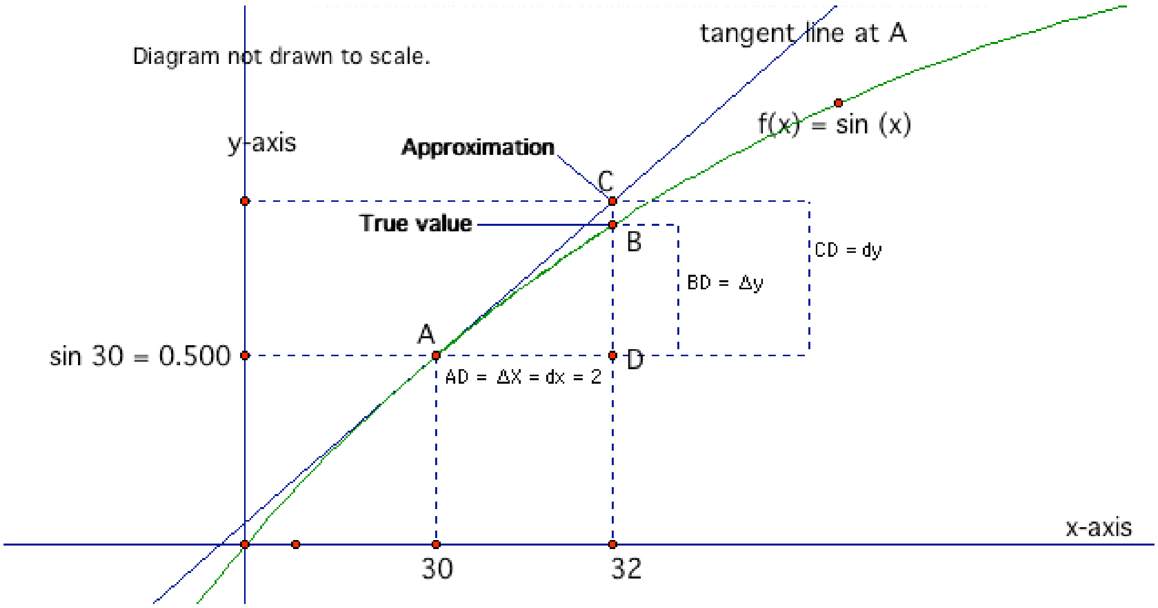


figure 4

$$f'(x) \approx \frac{\Delta y}{\Delta x} \to \Delta y \approx (\Delta x) f'(x)$$

Since $(a, f(a)) = (30°, \sin(30°)) = \left(\frac{\pi}{6} \, radians, \sin\left(\frac{\pi}{6}\right)\right)$ and $f'(x) = \cos(x)$,

$$\Delta y = (\Delta x) f'(x)$$

Then, $= \sin(32^\circ) - \sin(30^\circ) = (\sin(32^\circ) - \sin(30^\circ))\cos(30^\circ)$.

$$\approx \sin(\frac{32\pi}{180}) - \sin(\frac{\pi}{6}) = (\frac{32\pi}{180} - \frac{\pi}{6})\cos(\frac{\pi}{6}) \approx 0.0302$$

And so, $\sin(\frac{32\pi}{180}) \approx \sin(\frac{\pi}{6}) + 0.0302 \approx 0.500 + 0.0302 \approx 0.5302$.

Mathematical Focus 3

A ratio of measures of legs of a right triangle with an acute angle of measure x can be used to *approximate* sin(x). Because we cannot construct a right triangle with this angle measure, we need to use an alternative method to generate the triangle.

$Sin(32°)$ can be estimated by sketching a 32°-58°-90° right triangle with the aid of a protractor or software such as Geometer's Sketchpad, measuring the length of the hypotenuse and leg opposite the 32° angle, and computing the sine ratio.

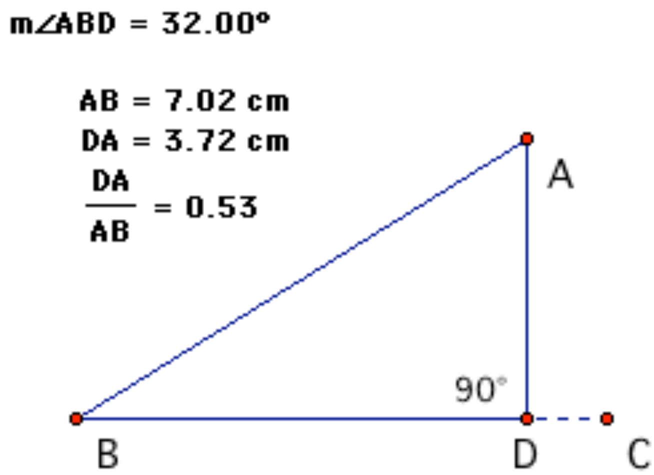


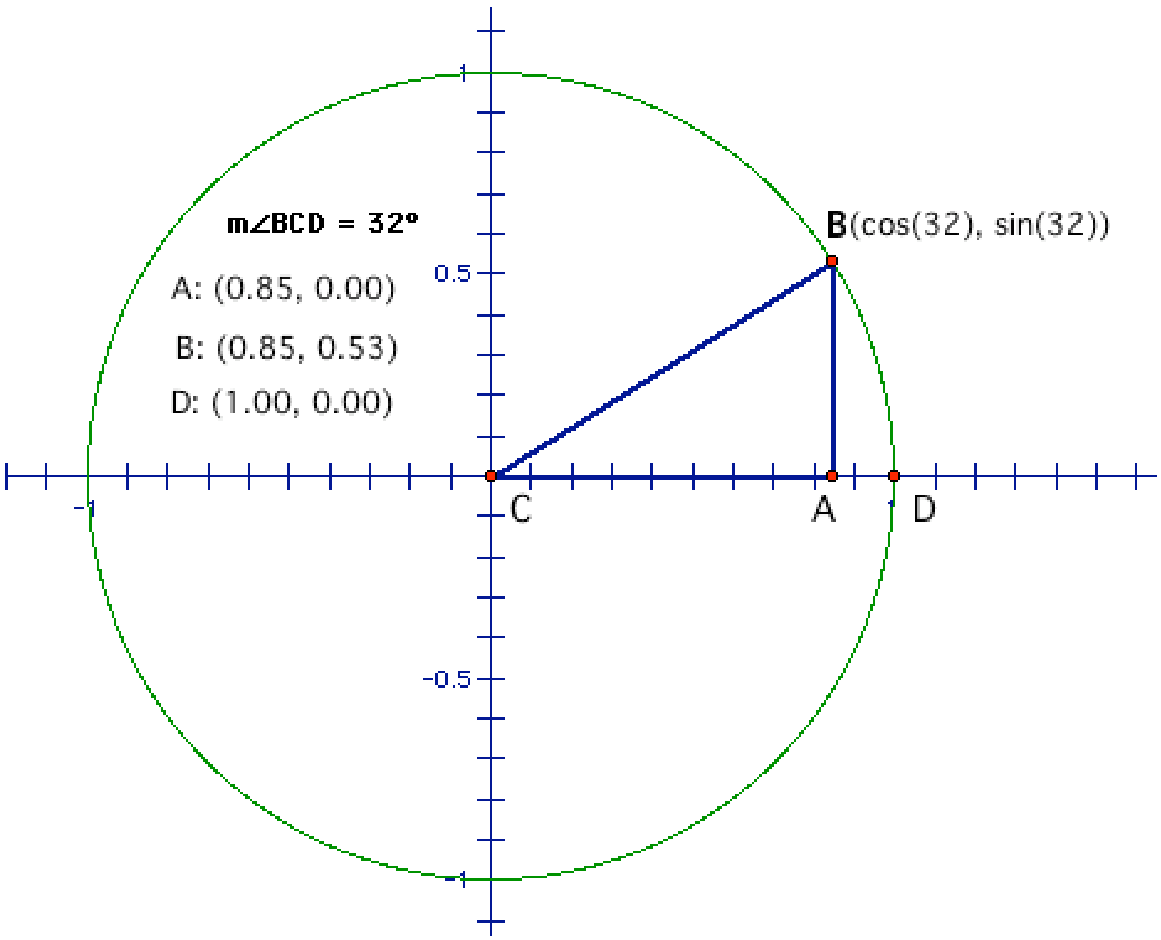
Figure 5

Mathematical Focus 3

The unit circle embodies both the sine function and the cosine function. For any point P on the unit circle, the distance from P to the horizontal axis is the sine of the angle in standard position formed by the ray from the origin to P and the x-axis. The coordinates of P are (cos(θ), sin(θ)).

Therefore, the distance from P to the x-axis is approximately equal to sin (32°). The measure of segment AB is approximately 0.53 and so, $\sin(32°) \approx 0.53$

Figure 6



References

None.